



Strand II: Geometry and Measurement

Standard I: Shape and Shape Relationships - Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes, identify properties and describe relationships among shapes.

Key Ideas:

- 1. Spatial sense relies on the ability to recognize and describe shape.
- 2. Recognizing attributes and characteristics of shapes is a prerequisite for understanding.
- 3. Comparing, sorting and classifying shapes leads to useful generalizations.
- 4. Drawing and constructing shapes in two and three dimensions are important ways to represent the world.
- 5. Understanding shapes requires recognition of what happens when shapes are combined, dissected or transformed.
- 6. Figures that are alike in size and/or shape and figures that have special relationships to each other lead to important generalizations.
- 7. Shape, shape properties, and shape relationships help students to describe and make sense of the physical world and to solve problems.

Middle School Benchmark	Grade 5	Grade 6	Grade 7	Grade 8
1. Distinguish among shapes and differentiate between examples and non-examples of shapes based on their properties; generalize about shapes of graphs and data distributions.				
2. Generalize the characteristics of shapes and apply their generalizations to classes of shapes.	Know the meaning of angles, and solve problems G.GS.05.02 Measure angles with a protractor and classify them as acute, right, obtuse or straight. G.GS.05.06 Understand why the sum of the interior angles of a triangle is 180° and the sum of the interior angles of a quadrilateral is 360°, and use these properties to solve problems.			Solve problems about geometric figures G.SR.08.03 Understand the definition of a circle; know and use the formulas for circumference and area of a circle to solve problems. G.SR.08.05 Solve applied problems involving areas of triangles, quadrilaterals, and circles. Understand concepts of volume and surface area, apply formulas G.SR.08.07 Understand the concept of surface area and find the surface area of prisms, cones, spheres, pyramids, and cylinders.
3. Derive generalizations about shapes and apply those generalizations to develop classifications of familiar shapes.				
4. Construct familiar shapes using coordinates, appropriate tools (including technology), sketching and drawing two- and three-dimensional shapes.	Understand the concept of volume M.TE.05.08 Build solids with unit cubes and state their volumes.	Find volume and surface area M.PS.06.02 Draw patterns (of faces) for a cube and rectangular prism that, when cut, will cover the solid exactly (nets). Construct geometric shapes G.SR.06.05 Use paper folding to perform basic geometric constructions of perpendicular lines, midpoints of line segments and angle bisectors, and justify informally.	Draw and construct geometric objects G.SR.07.01 Use a ruler and other tools to draw squares, rectangles, triangles and parallelograms with specified dimensions.	Visualize solids G.SR.08.08 Sketch a variety of two-dimensional representations of three-dimensional solids, including: orthogonal views (top, front, and side), picture views (projective or isometric) and nets, and use such two-dimensional representations to help solve problems.

5. Combine, dissect and transform shapes.		Understand the concept of congruence and basic transformations G.TR.06.03 Understand the basic rigid motions in the plane (reflections, rotations, translations), relate these to congruence, and apply them to solve problems. G.TR.06.04 Understand and use simple compositions of basic rigid transformations, e.g., a translation followed by a reflection.	Draw and construct geometric objects G.SR.07.02 Use compass and straightedge to perform basic geometric constructions: the perpendicular bisector of a segment, an equilateral triangle, and the bisector of an angle; understand informal justifications.	Solve problems about geometric figures G.SR.08.04 Find area and perimeter of complex figures by subdividing them into basic shapes (quadrilaterals, triangles, circles).
6. Generalize about the common properties of similar, congruent, parallel and perpendicular shapes and verify their generalizations informally.		Understand and apply basic properties G.GS.06.01 Understand and apply basic properties of lines, angles, and triangles, including: <ul style="list-style-type: none">• triangle inequality• relationships of vertical angles, complementary angles, supplementary angles• congruence of corresponding and alternate interior angles when parallel lines are cut by a transversal, and that such congruencies imply parallel lines• locate interior and exterior angles of any triangle and use the property that a exterior angle of a triangle is equal to the sum of the remote (opposite) interior angles know that the sum of the exterior angles of a convex polygon is 360° Understand the concept of congruence and basic transformations G.GS.06.02 Understand that for polygons, congruence means corresponding sides and angles have equal measures.		Understand and apply concepts of transformation and symmetry G.TR.08.09 Understand the definition of a dilation from a point in the plane and relate it to the definition of similar polygons. G.TR.08.10 Understand and use reflective and rotational symmetries of two- dimensional shapes and relate them to transformations to solve problems.
7. Use shape, shape properties and shape relationships to describe the physical world and to solve problems.			Understand and apply directly proportional relationships; relate to linear relationships A.PA.07.04 For directly proportional or linear situations, solve applied problems using graphs and equations; e.g., the heights and volume of a container with uniform cross-section; height of water in a tank being filled at a constant rate; degrees Celsius and degrees Fahrenheit; distance and time under constant speed. A.PA.07.05 Understand and use directly proportional relationships of the form $y = mx$, and distinguish from linear relationships of the form $y = mx + b$, b non-zero; understand that in a directly proportional relationship between two quantities one quantity is a constant multiple of the other quantity.	Understand and use the Pythagorean Theorem G.GS.08.01 Understand at least one proof of the Pythagorean Theorem; use the Pythagorean Theorem and its converse to solve applied problems, including perimeter, area, and volume problems. Solve problems about geometric figures G.SR.08.05 Solve applied problems involving areas of triangles, quadrilaterals, and circles. Visualize solids G.SR.08.08 Sketch a variety of two-dimensional representations of three-dimensional solids, including: orthogonal views (top, front, and side), picture views (projective or isometric) and nets, and use such two-dimensional representations to help solve problems.



Strand II: Geometry and Measurement

Standard 2: Position - Students identify locations of objects, identify location relative to other objects, and describe the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object.

Key Ideas:

- 1. Locating physical objects or points in space requires understanding of position.
- 2. Concepts of direction, orientation, relative position and symmetry enable students to describe objects relative to their surroundings.
- 3. Certain actions can change the size, shape, position or orientation of an object.
- 4. Locating all the points that satisfy a condition or the special points that satisfy two or more conditions is an important spatial ability.
- 5. Concepts of position, direction and orientation enable students to describe the physical world and to solve problems.

Middle School Benchmark	Grade 5	Grade 6	Grade 7	Grade 8
1. Locate and describe objects in terms of their position, including compass directions, Cartesian coordinates, latitude and longitude and midpoints.				
2. Locate and describe objects in terms of their orientation and relative position, including coincident, collinear, parallel, perpendicular; differentiate between fixed (e.g., N- S- E- W) and relative (e.g., right-left) orientations; recognize and describe examples of bilateral and rotational symmetry.		Construct geometric shapes G.SR.06.05 Use paper folding to perform basic geometric constructions of perpendicular lines, midpoints of line segments and angle bisectors, and justify informally.		Understand and apply concepts of transformation and symmetry G.TR.08.10 Understand and use reflective and rotational symmetries of two- dimensional shapes and relate them to transformations to solve problems.
3. Describe translations, reflections, rotations and dilations using the language of transformations and employ transformations to verify congruence of figures.	Know the meaning of angles, and solve problems G.TR.05.01 Associate an angle with a certain amount of turning; know that angles are measured in degrees; understand that 90°, 180°, 270°, and 360° are associated, respectively, with 1/4, 1/2, 3/4 and full turns.	Understand the concept of congruence and basic transformations G.TR.06.03 Understand the basic rigid motions in the plane (reflections, rotations, translations), relate these to congruence, and apply them to solve problems. G.TR.06.04 Understand and use simple compositions of basic rigid transformations, e.g., a translation followed by a reflection.		Understand and apply concepts of transformation and symmetry G.TR.08.09 Understand the definition of a dilation from a point in the plane and relate it to the definition of similar polygons. G.TR.08.10 Understand and use reflective and rotational symmetries of two- dimensional shapes and relate them to transformations to solve problems.
4. Locate the position of points or objects described by two or more conditions; locate all the points (locus) that satisfy a given condition.		Understand the coordinate plane A.RP.06.02 Plot ordered pairs of integers and use ordered pairs of integers to identify points in all four quadrants of the coordinate plane.		Understand and use the Pythagorean Theorem G.LO.08.02 Find the distance between two points on the coordinate plane, using the distance formula; recognize that the distance formula is an application of the Pythagorean Theorem.
5. Use concepts of position, direction and orientation to describe the physical world and to solve problems.		Understand the concept of congruence and basic transformations G.TR.06.03 Understand the basic rigid motions in the plane (reflections, rotations, translations), relate these to congruence, and apply them to solve problems.		



II: Geometry and Measurement

Standard 3: Measurement - Students compare attributes of two objects, or of one object with a standard (unit), and analyze situations to determine what measurement(s) should be made and to what level of precision.

- Key Ideas:
- 1. A fundamental component of measurement and learning to measure is the comparison of an object or property to a unit of comparison
 - 2. Measurement requires that students identify the attribute to be measured and an appropriate unit.
 - 3. Students develop a better understanding of the physical world if they regularly estimate before they measure and evaluate their estimates after they measure.
 - 4. Measurement is incomplete unless students interpret the meaning and significance of their results.
 - 5. It is not always possible to measure a quantity directly; in such cases students must use other indirect means.
 - 6. Measurement reflects the usefulness and practicality of mathematics and puts students in touch with the physical world.

Middle School Benchmark	Grade 5	Grade 6	Grade 7	Grade 8
1. Select and use appropriate tools; measure objects using standard units in both the metric and common systems and measure angles in degrees.	Understand the concept of volume M.TE.05.09 Use filling (unit cubes or liquid), and counting or measuring to find the volume of a cube and rectangular prism. Know the meaning of angles, and solve problems G.TR.05.01 Associate an angle with a certain amount of turning; know that angles are measured in degrees; understand that 90°, 180°, 270°, and 360° are associated, respectively, with 1/4, 1/2, 3/4 and full turns. G.GS.05.02 Measure angles with a protractor and classify them as acute, right, obtuse or straight.		Draw and construct geometric objects G.SR.07.01 Use a ruler and other tools to draw squares, rectangles, triangles and parallelograms with specified dimensions. G.SR.07.02 Use compass and straightedge to perform basic geometric constructions: the perpendicular bisector of a segment, an equilateral triangle, and the bisector of an angle; understand informal justifications.	
2. Identify the attribute to be measured and select the appropriate unit of measurement for length, mass (weight), time, temperature, perimeter, area, volume, and angle.	Know, and convert among, measurement units within a given system M.UN.05.01 Recognize the equivalence of 1 liter, 1000 ml and 1000 cm ³ and include conversions among liters, milliliters, and cubic centimeters. M.UN.05.02 Know the units of measure of volume: cubic centimeter, cubic meter, cubic inches, cubic feet, cubic yards, and use their abbreviations: cm ³ , m ³ , in ³ , ft ³ , yd ³ . M.UN.05.03 Compare the relative sizes of one cubic inch to one cubic foot, and one cubic centimeter to one cubic meter.			
3. Estimate measures with a specified degree of accuracy and decide if an estimate or a measurement is “close enough.”	.			

4. Interpret measurements and recognize that two objects may have the same measurement on one attribute (e.g., area) but not necessarily on another (e.g., perimeter).	Find areas of geometric shapes using formulas M.PS.05.05 Represent relationships between areas of rectangles, triangles and parallelograms using models.		Understand the concept of similar polygons, and solve related problems G.TR.07.06 Understand and use the fact that when two triangles are similar with scale factor of r, their areas are related by a factor of r ² .	
5. Use proportional reasoning and indirect measurements to draw inferences.	Know, and convert among, measurement units within a given system M.UN.05.01 Recognize the equivalence of 1 liter, 1000 ml and 1000 cm ³ and include conversions among liters, milliliters, and cubic centimeters. Know, and convert among, measurement units within a given system M.UN.05.04 Convert measurements of length, weight, area, volume, and time within a given system, using easily manipulated numbers. Find areas of geometric shapes using formulas M.TE.05.06 Understand and know how to use the area formula of a triangle: $A = \frac{1}{2}bh$ (where b is length of the base and h is the height), and represent using models and manipulatives. M.TE.05.07 Understand and know how to use the area formula for a parallelogram: $A = bh$, and represent using models and manipulatives. Know the meaning of angles, and solve problems G.GS.05.03 Identify and name angles on a straight line and vertical angles. G.GS.05.04 Find unknown angles in problems involving angles on a straight line, angles surrounding a point and vertical angles. G.GS.05.05 Know that angles on a straight line add up to 180° and angles surrounding a point add up to 360°; justify informally by “surrounding” a point with angles. Solve problems about geometric shapes G.GS.05.07 Find unknown angles using the properties of: triangles, including right, isosceles, and equilateral triangles; parallelograms, including rectangles and rhombuses; and trapezoids.	Convert within measurement systems M.UN.06.01 Convert between basic units of measurement within a single measurement system, e.g., square inches to square feet. Find volume and surface area M.TE.06.03 Compute the volume and surface area of cubes and rectangular prisms given the lengths of their sides, using formulas.	Understand the concept of similar polygons, and solve related problems G.TR.07.03 Understand that in similar polygons, corresponding angles are congruent and the ratios of corresponding sides are equal; understand the concepts of similar figures and scale factor. G.TR.07.04 Solve problems about similar figures and scale drawings. G.TR.07.05 Show that two triangles are similar using the criteria: corresponding angles are congruent (AAA similarity); the ratios of two pairs of corresponding sides are equal and the included angles are congruent (SAS similarity); ratios of all pairs of corresponding sides are equal (SSS similarity); use these criteria to solve problems and to justify arguments. G.TR.07.06 Understand and use the fact that when two triangles are similar with scale factor of r, their areas are related by a factor of r ² . Understand and solve problems involving rates, ratios, and proportions N.FL.07.05 Solve simple proportion problems using such methods as unit rate, scaling, finding equivalent fractions, and solving the proportion equation $a/b = c/d$; know how to see patterns about proportional situations in tables.	Solve problems about geometric figures G.SR.08.03 Understand the definition of a circle; know and use the formulas for circumference and area of a circle to solve problems. G.SR.08.04 Find area and perimeter of complex figures by subdividing them into basic shapes (quadrilaterals, triangles, circles). G.SR.08.05 Solve applied problems involving areas of triangles, quadrilaterals, and circles. Understand concepts of volume and surface area, apply formulas G.SR.08.06 Know the volume formulas for generalized cylinders ((area of base) x height), generalized cones and pyramids ($\frac{1}{3}$ (area of base) x height) and spheres ($\frac{4}{3}\pi$ (radius) ³), and apply them to solve problems. G.SR.08.07 Understand the concept of surface area and find the surface area of prisms, cones, spheres, pyramids, and cylinders.
6. Apply measurement to describe the real world and to solve problems.	Understand the concept of volume M.PS.05.10 Solve applied problems about the volumes of rectangular prisms using multiplication and division and using the appropriate units.			